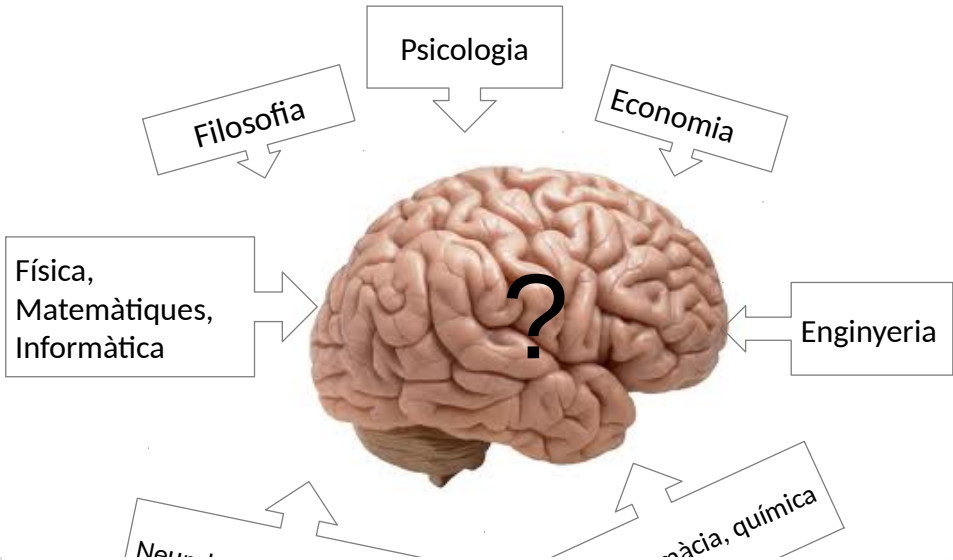


Matemàtiques per entendre el cervell

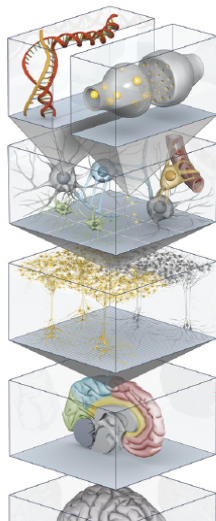
Gemma Huguet
Universitat Politècnica de Catalunya

13-3-2021

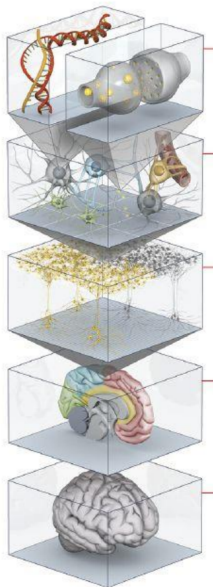




El cervell



El cervell en diferents escales



Molecular

Cel·lular

Circuits

Regions

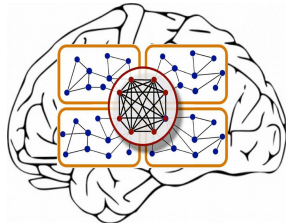
Òrgan



Eines matemàtiques en Neurociència



Data analysis, statistics,
information theory

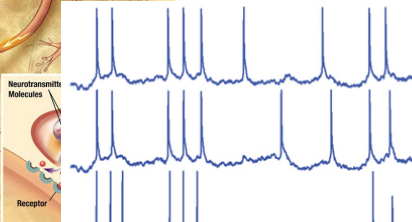
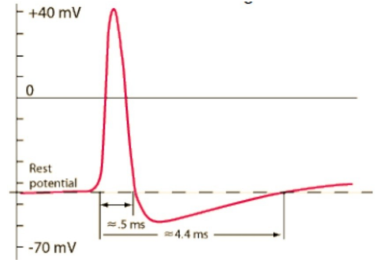
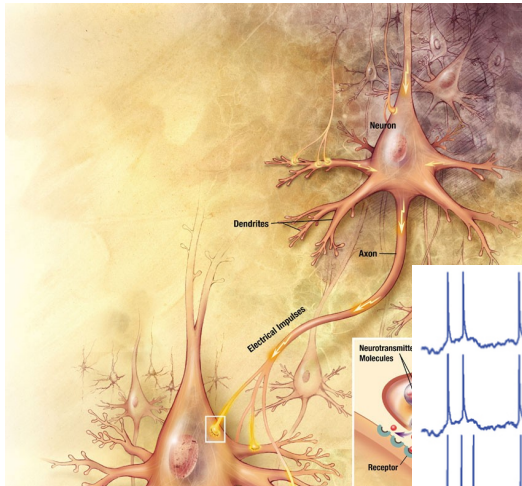


Graph theory, topology

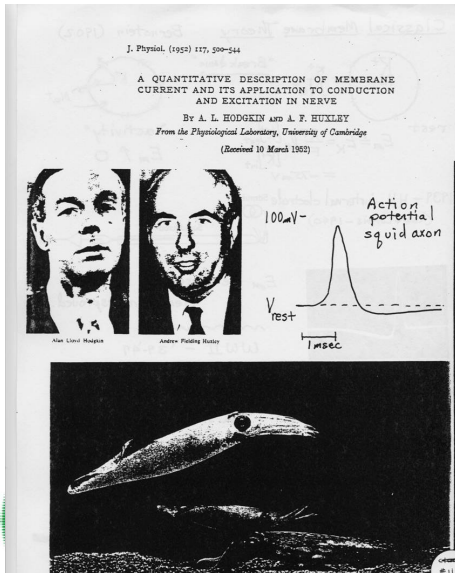
$$\begin{cases} C\dot{V} &= -I_{Na} - I_K - I_L + I_{app} \\ &= -g_{Na}m^3h(V - V_{Na}) - g_Kn^4(V - V_K) - g_L(V - V_L) + I_{app} \\ \dot{m} &= (m_\infty(V) - m)/\tau_m(V) \\ \dot{h} &= (h_\infty(V) - h)/\tau_h(V) \\ \dot{n} &= (n_\infty(V) - n)/\tau_n(V) \end{cases}$$

Continuous models: differential equations,
dynamical systems, stochastic processes

La neurona



Model de Hodgkin-Huxley (1952)



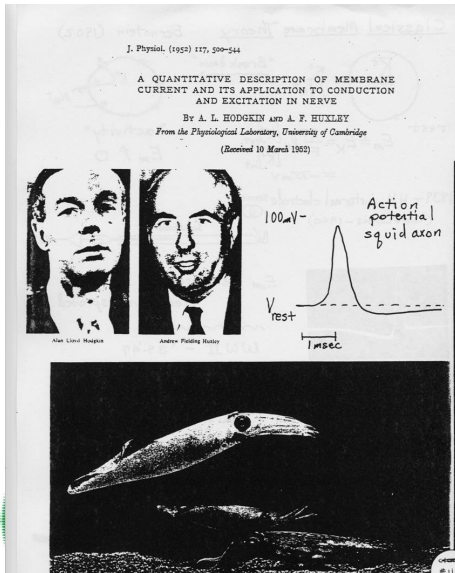
$$C \frac{dV}{dt} = -g_{Na} m^3 h (V - V_{Na}) - g_K n^4 (V - V_K) - g_L (V - V_L) + I$$

$$\frac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V)m$$

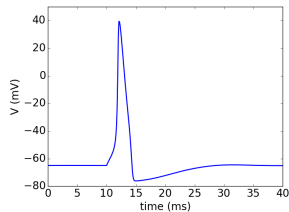
$$\frac{dh}{dt} = \alpha_h(V)(1-h) - \beta_h(V)h$$

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$

Model de Hodgkin-Huxley (1952)

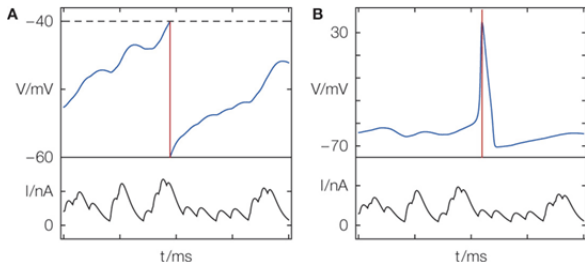


$$C \frac{dV}{dt} = -g_{Na} m^3 h (V - V_{Na}) - g_K n^4 (V - V_K) - g_L (V - V_L) + I$$
$$\frac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V)m$$
$$\frac{dh}{dt} = \alpha_h(V)(1-h) - \beta_h(V)h$$
$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$



Model *Integrate-and-fire*

$$\left\{ \begin{array}{l} C\dot{V} = -g_L(V - E_L) + I_{app}, \\ \text{si } V(t) > V_{th}, \text{ aleshores } V \rightarrow V_{re} \end{array} \right.$$



- *Integrate-and-fire* (forma adimensional):

$$v' = -v + I, \text{ si } v(t) = 1, \text{ aleshores } v(t^+) = 0$$



- *Integrate-and-fire* (forma adimensional):

$$v' = -v + I, \text{ si } v(t) = 1, \text{ aleshores } v(t^+) = 0$$

- Solució: $v(t) = I + (v_0 - I)e^{-t}$, amb $v(0) = v_0$.

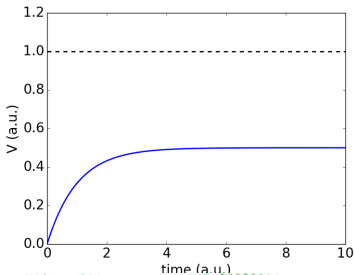


- *Integrate-and-fire* (forma adimensional):

$$v' = -v + I, \text{ si } v(t) = 1, \text{ aleshores } v(t^+) = 0$$

- Solució: $v(t) = I + (v_0 - I)e^{-t}$, amb $v(0) = v_0$.

$$I = 0.5, v_0 = 0$$

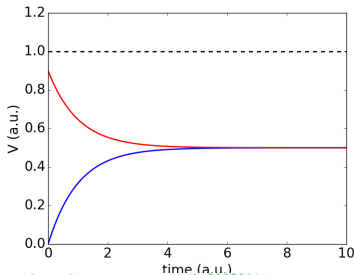


- *Integrate-and-fire* (forma adimensional):

$$v' = -v + I, \text{ si } v(t) = 1, \text{ aleshores } v(t^+) = 0$$

- Solució: $v(t) = I + (v_0 - I)e^{-t}$, amb $v(0) = v_0$.

$$I = 0.5, v_0 = 0 \quad v_0 = 0.9$$

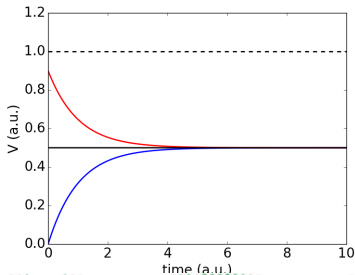


- *Integrate-and-fire* (forma adimensional):

$$v' = -v + I, \text{ si } v(t) = 1, \text{ aleshores } v(t^+) = 0$$

- Solució: $v(t) = I + (v_0 - I)e^{-t}$, amb $v(0) = v_0$.

$$I = 0.5, \quad v_0 = 0 \quad v_0 = 0.9 \quad v_0 = 0.5.$$



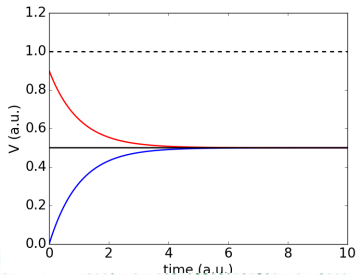
- *Integrate-and-fire* (forma adimensional):

$$v' = -v + I, \text{ si } v(t) = 1, \text{ aleshores } v(t^+) = 0$$

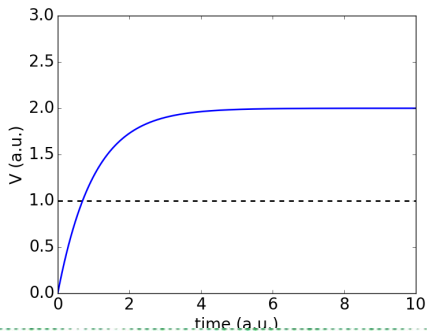
- Solució: $v(t) = I + (v_0 - I)e^{-t}$, amb $v(0) = v_0$.

$$I = 0.5, v_0 = 0 \quad v_0 = 0.9 \quad v_0 = 0.5.$$

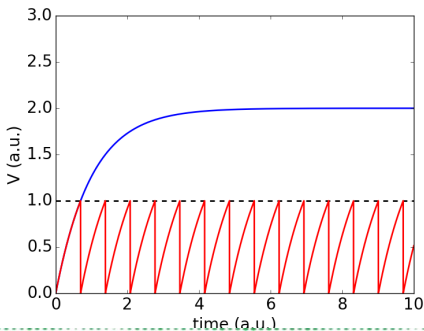
$v = I = 0.5$ és un punt d'equilibri estable



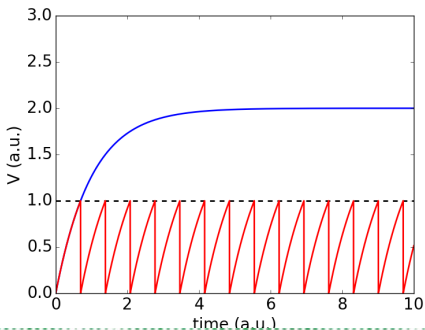
- Considerem $I = 2$, $v_0 = 0$.



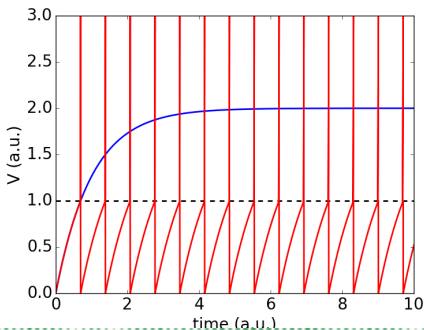
- Considerem $I = 2$, $v_0 = 0$.
- Afegim la reinicialització



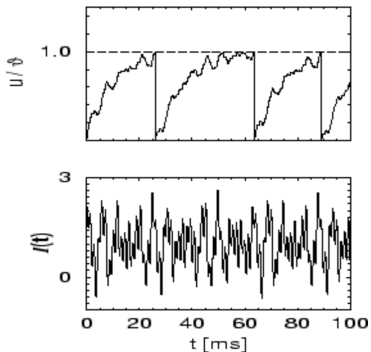
- Considerem $I = 2$, $v_0 = 0$.
- Afegim la reinicialització
- Canvi qualitatiu en les solucions (punt equilibri estable \rightarrow solució periòdica estable).
Bifurcació.



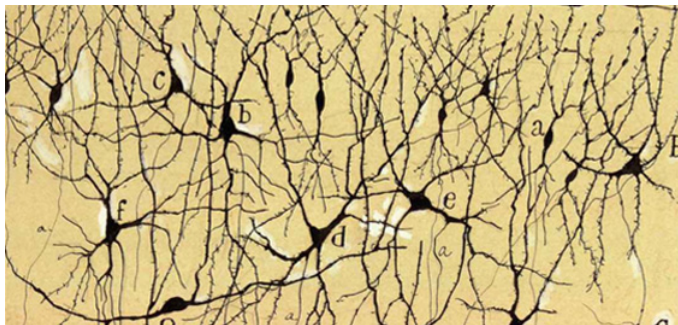
- Considerem $I = 2$, $v_0 = 0$.
- Afegim la reinicialització
- Canvi qualitatiu en les solucions (punt equilibri estable \rightarrow solució periòdica estable).
Bifurcació.



Senyal d'entrada dependent del temps

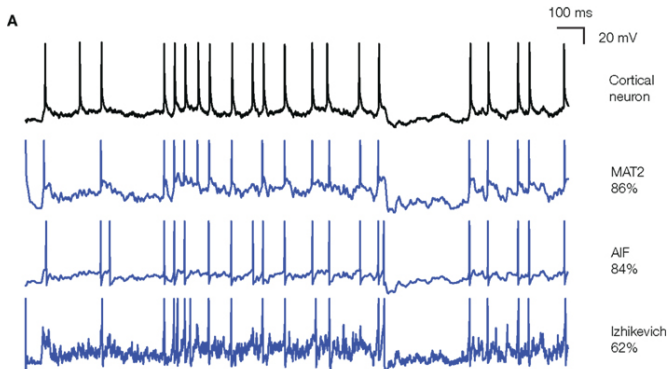


Circuits Neuronals

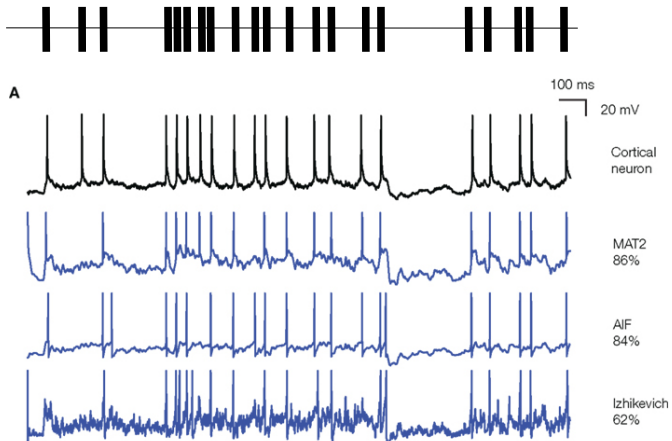


Dibuix de S. Ramon y Cajal
Sinapsis Neuronals: Excitadores i Inhibidores

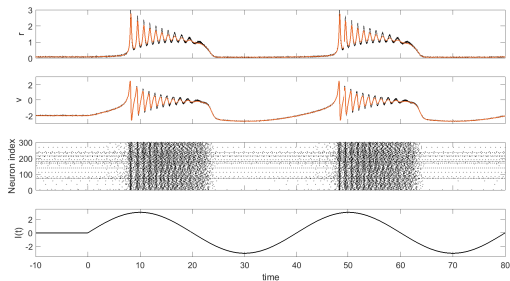
Simulacions



Simulacions

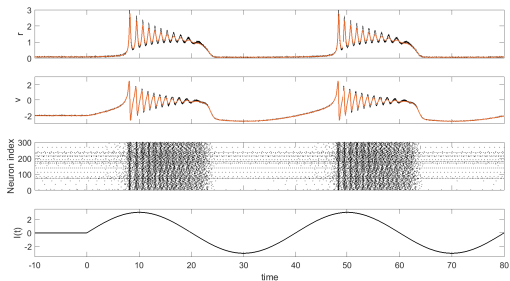


Dibuix de trama (*raster plot*) i Freqüència de Descàrrega



Simulacions de D. Reyner basades en Montbrí et al 2015.

Dibuix de trama (*raster plot*) i Freqüència de Descàrrega

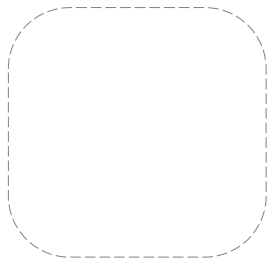
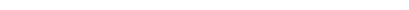


Simulacions de D. Reyner basades en Montbrío et al 2015.

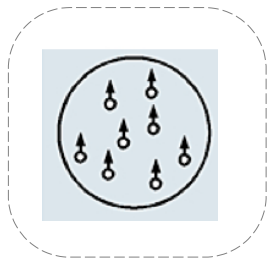
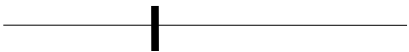
Models de camp mitjà: Equacions de Wilson-Cowan.



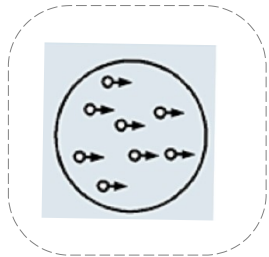
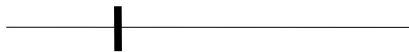
Codi Neuronal



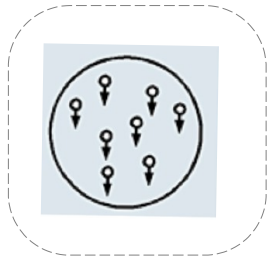
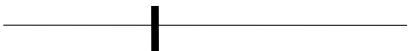
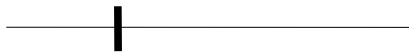
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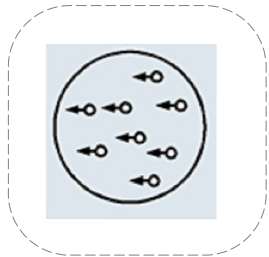
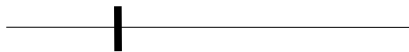
Codi Neuronal



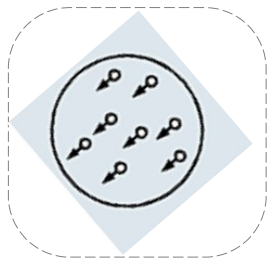
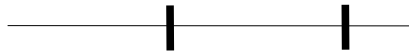
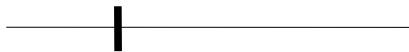
Codi Neuronal



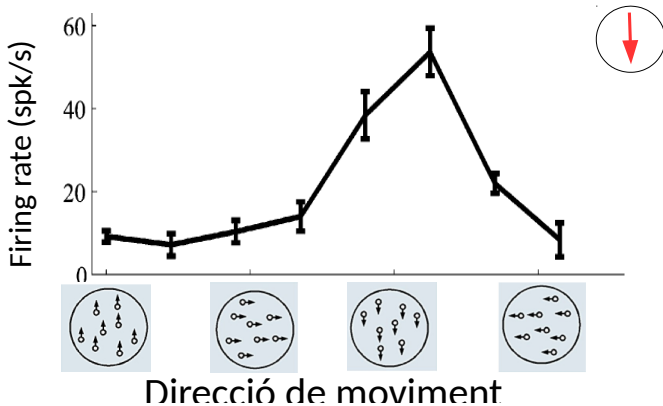
Codi Neuronal



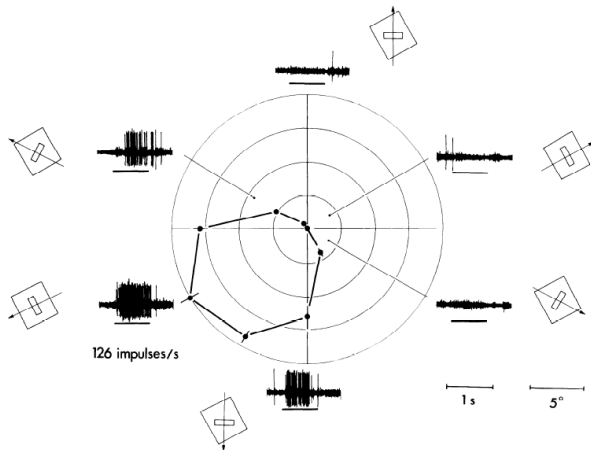
Codi Neuronal



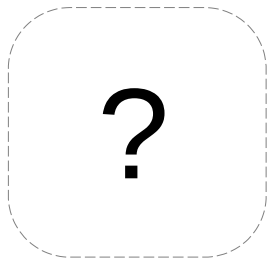
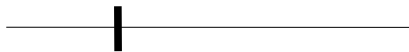
Codi Neuronal



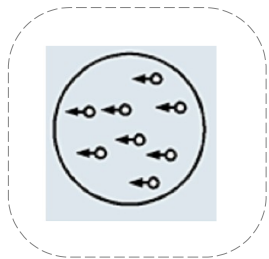
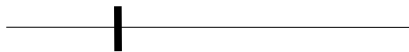
Experiments area MT del còrtex visual



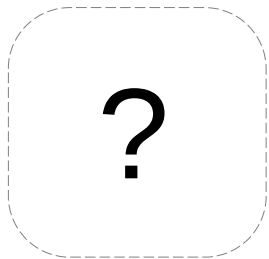
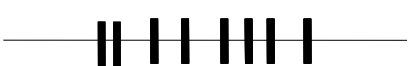
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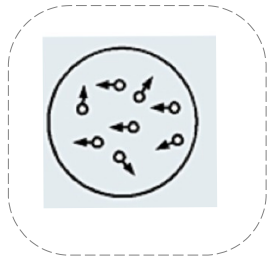
Codi Neuronal



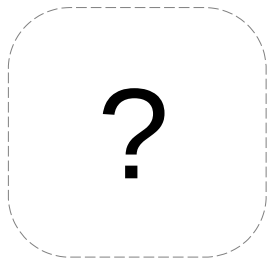
Codi Neuronal



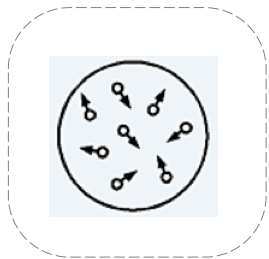
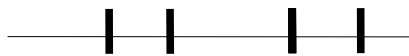
Codi Neuronal



Codi Neuronal



Codi Neuronal



Motion coherence and MT neurons

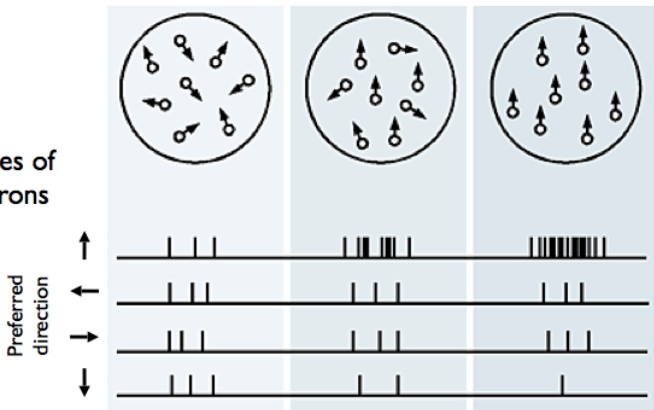
Motion stimulus

no coherence

50% coherence

100% coherence

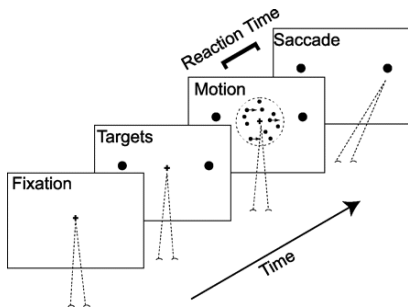
Responses of
MT neurons



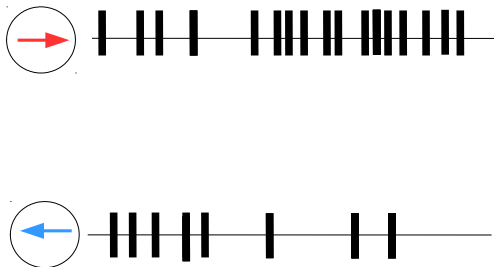
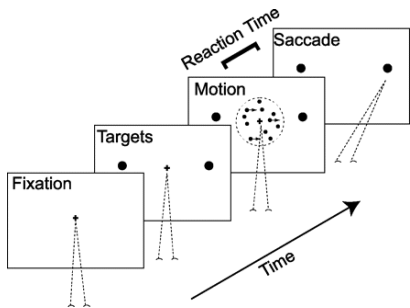


JORNADA SCM • Matemàtiques per a un món millor #idm314cat #MatemàtiquesMónMillor

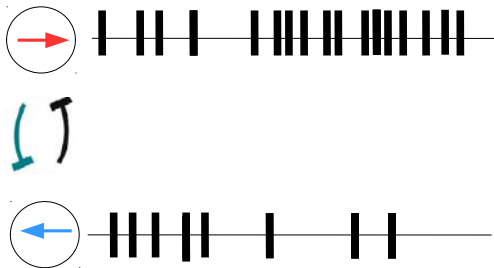
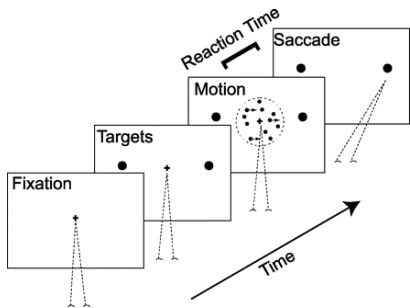
Decisions perceptual



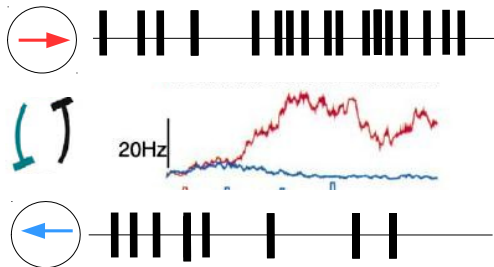
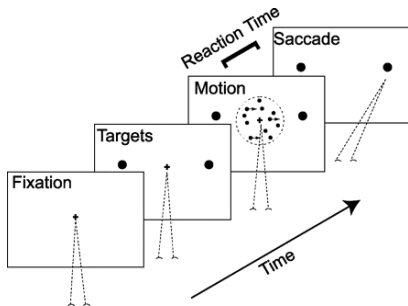
Decisions perceptual



Decisions perceptual

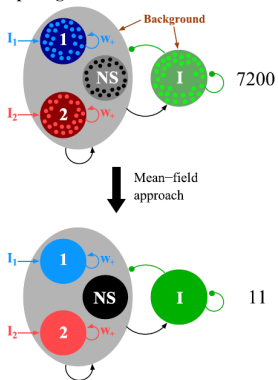


Decisions perceptual



El model

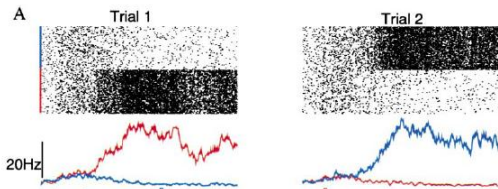
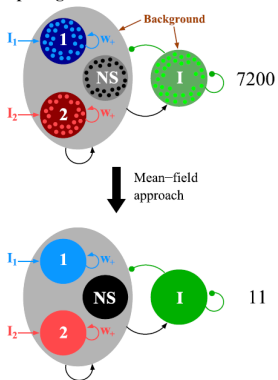
Spiking neuronal network model



Wang 2002, Wong and Wang, 2002

El model

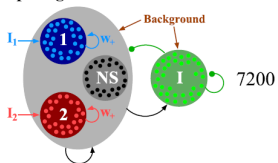
Spiking neuronal network model



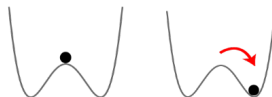
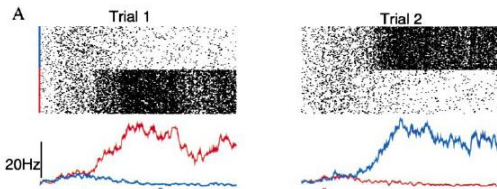
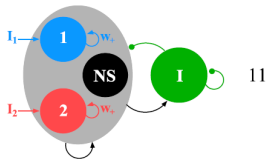
Wang 2002, Wong and Wang, 2002

El model

Spiking neuronal network model



Mean-field approach



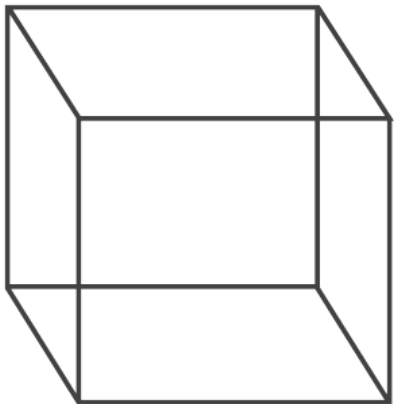
Wang 2002, Wong and Wang, 2002



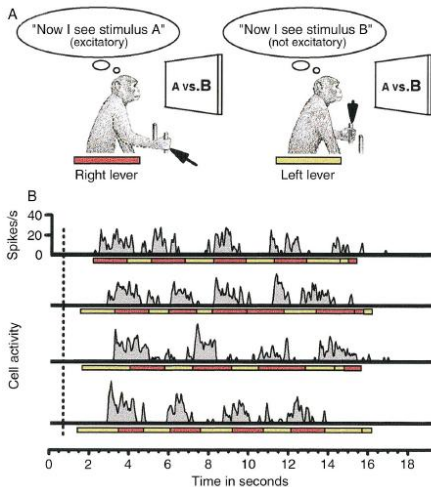
JORNADA SCM • Matemàtiques per a un món millor

#idm314cat #MatemàtiquesMónMillor

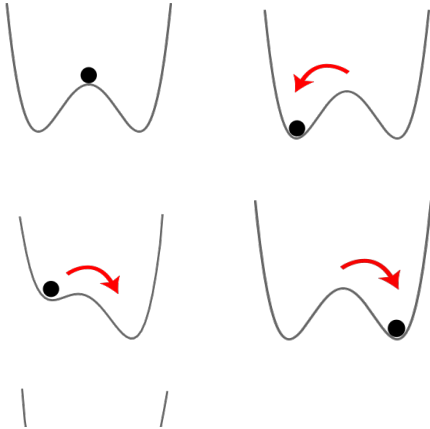
Figures ambigües



Què passa al nostre cervell?



El model



$$\begin{aligned} \mathbb{E} \frac{dr_1}{dt} &= -r_1 + f(-\beta r_2 - \gamma a_1 + I_1) \\ \mathbb{E} \frac{dr_2}{dt} &= -r_2 + f(-\beta r_1 - \gamma a_2 + I_2) \end{aligned}$$

Per què un model del cervell?

- Per a reproduir-lo \rightarrow nous algoritmes
- Per a curar-lo \rightarrow noves teràpies
- Per a comprendre'l millor \rightarrow nou coneixement



ENTREVISTA

SOCIETAT | 12/12/2020

Fernando Giráldez: “La comprensió profunda del cervell vindrà de les matemàtiques”

Entrevista al catedràtic de la Universitat Pompeu Fabra

Toni Pou

5 min



Fernando Giráldez: “La comprensió profunda del cervell vindrà de les matemàtiques” F. G.

Moltes gràcies per la vostra atenció

